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String-derived MSSM vacua with residual R symmetries

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Abstract

Recently it was shown that there is a unique $\mathbb{Z}_4^R$ symmetry for the MSSM which allows the Yukawa couplings and dimension five neutrino mass operator, forbids the μ term and commutes with SO(10). This $\mathbb{Z}_4^R$ symmetry contains matter parity as a subgroup and forbids dimension four and five proton decay operators. We show how to construct string vacua with discrete R symmetries in general and this symmetry in particular, and present an explicit example which exhibits the exact MSSM spectrum, the $\mathbb{Z}_4^R$ symmetry as well as other desired features such as gauge-top unification. We introduce the Hilbert basis method for determining all D-flat configurations and efficient algorithms for identifying field configurations with a desired residual symmetry. These methods are used in an explicit example, in which we describe in detail how to construct a supersymmetric vacuum configuration with the phenomenologically attractive $\mathbb{Z}_4^R$ symmetry. At the perturbative level, this is a supersymmetric Minkowski vacuum in which almost all singlet fields (moduli) are fixed.

1 Introduction

There are many independent observations hinting at the relevance of a high scale for particle physics. The smallness of neutrino masses has a simple explanation in terms of the see-saw mechanism [1] and relies on the existence of heavy singlet neutrinos. Stabilizing the electroweak scale against the see-saw scale seems to require supersymmetry; remarkably, the simplest supersymmetric extension of the standard model, the MSSM, realizes the compelling scenario of gauge unification [2] at a scale $M_{\mathrm{GUT}} \simeq 2 \cdot 10^{16}$ GeV, which is suspiciously close to the see-saw scale. Both scales are not too far from $M_{\mathrm{P}} \simeq 2{\cdot}10^{18}$ GeV, which is set by Newton's constant.

The question of how to incorporate all scales in a coherent scheme has been addressed for more than 30 years. From a bottom-up perspective one is led to the scheme of grand unified theories (GUTs). Although this scheme exhibits various very appealing features, there are three major obstacles. First, there is the so-called doublet-triplet splitting, and related to it, the MSSM μ problem. Second, even if this problem is solved, unified models typically are in conflict with dimension five proton decay [3, 4] operators. (It is well known that dimension four proton decay can be forbidden by matter parity.[1]) Third, in four-dimensional models of grand unification there is no relation between the GUT and Planck scales, M_{GUT} and M_{P}.

String theory is believed to provide us with such a relation. However, if string theory is to describe the real world, it should also provide us with solutions to the first and second problems. In fact, as known for a long time, the doublet-triplet splitting problem has a simple solution in theories with extra dimensions in which the GUT symmetry is broken in the process of compactification [5, 6], which also avoids the most stringent problems with dimension five proton decay [7]. However, in concrete string compactifications (see [8–10] for early attempts and [11, 12] for a different approach) very often the problem is reintroduced; this applies also to the models discussed more recently [13–16]. On the other hand, one cannot rule out these constructions as their vacua are not completely understood. That is, the analysis of potentially realistic string models is a non-trivial task since a given model exhibits a plethora of vacua with very different features. The role of discrete symmetries in identifying and analyzing such vacua has been stressed recently [17]. One of these symmetries is matter parity, which has been successfully embedded in string theory [16]. This study is devoted to a discussion of the role of further discrete symmetries in such models and their phenomenological implications. Specifically, we will focus on a proposed $\mathbb{Z}_4^R$ symmetry [18] which has recently been shown in [19] to be the unique anomaly-free possibility with the following properties:

1. it forbids the μ term at the perturbative level;

2. it allows the MSSM Yukawa couplings and the effective neutrino mass operator;

[1]Matter parity is sometimes also known as "R parity". We choose not to use this terminology as matter parity is non-R, i.e. it commutes with supersymmetry, and one point of this paper is to discuss a true discrete R symmetry.

3. it commutes with SO(10) in the matter sector.

This symmmetry has the appealing feature that it forbids automatically dimension four and five proton decay operators. We will discuss how to identify string vacua exhibiting this symmetry and present a globally consistent string-derived model with the exact MSSM spectrum realizing this symmetry.

We start in section 2 with a short description of the general picture. In section 3 we present an explicit string-derived model in which an anomalous $\mathbb{Z}_4^R$ symmetry explains a suppressed vacuum expectation value of the superpotential, provides us with a solution of the μ problem and suppresses dimension five proton decay operators. Section 4 contains our conclusions. In various appendices we collect details of our calculations.

2 General picture

String theory compactifications provide us with a plethora of vacuum configurations, each of which comes with symmetries and, as a consequence, with extra massless degrees of freedom whose mass terms are prohibited by these symmetries. Simple examples for such compactifications include heterotic orbifolds [20, 21], where the rank of the gauge group after compactification equals that of $\mathrm{E}_8 \times \mathrm{E}_8$, i.e. 16. A few hundreds of orbifold models are known in which $\mathrm{E}_8 \times \mathrm{E}_8$ gets broken to the standard model gauge symmetry $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ (with hypercharge in GUT normalization) times $\mathrm{U}(1)^n$ times a hidden sector group and the chiral spectra of the MSSM [15, 22]. They also exhibit exotics which are vector-like with respect to G_{SM} and which can be decoupled when the extra gauge symmetries are broken. Each of these models contains many vacua, i.e. solutions of the supersymmetry conditions $V_F = V_D = 0$. Typically these vacua exhibit flat directions before supersymmetry breaking.

At the orbifold point, where the vacuum expectation values (VEVs) of all fields are zero, we have discrete R as well as continuous and discrete non-R symmetries. Typically one of the U(1) symmetries appears anomalous, which is conventionally denoted by $\mathrm{U}(1)_{\mathrm{anom}}$. Also some of the discrete symmetries may appear anomalous [23, 24]. After assigning VEVs to certain fields, some of the symmetries are spontaneously broken and others remain. We shall be mainly interested in remnant discrete symmetries, which can be of R or non-R type and be either anomalous or non-anomalous. We will discuss examples of all kinds in section 3.

Clearly, one cannot assign VEVs to the fields at will. Rather, one has to identify field configurations which correspond to local minima of the (effective) scalar potential. Let us briefly describe the first steps towards identifying such vacuum configurations. Consider a configuration in which several fields attain VEVs. We focus on "maximal vacua" (as in [17]), i.e. we assume that all fields which are neutral under the remnant gauge and discrete symmetries, called $\phi^{(i)}$ $(1 \leq i \leq N)$ in what follows, attain VEVs (if these are consistent with D-flatness). All fields without expectation value, denoted by $\psi^{(j)}$ $(1 \leq j \leq M)$, therefore transform non-trivially under some of the remnant symmetries.

2.1 Discrete non-R symmetries

The case of vacua with non-R discrete symmetries has been discussed in detail in [14,17]. In this case, the superpotential has the form

$$\mathscr{W} \;=\; \Omega(\phi^{(1)}, \dots \phi^{(N)}) + (\text{terms at least quadratic in the } \psi^{(j)}) \;. \tag{2.1}$$

Therefore, the F-term equations for the $\psi^{(j)}$ fields trivially vanish and we are left with N F-term equations for the N $\phi^{(i)}$ fields, which generically have solutions. Hence, if all $\phi^{(i)}$ enter gauge invariant monomials composed of $\phi^{(i)}$ fields only, we will find supersymmetric vacua, i.e. solutions to the F- and D-term equations.

Because of the above arguments it is sufficient to look at the system of $\phi^{(i)}$ fields only, which has been studied in the literature. Consider the case of a *generic* superpotential $\mathscr{W}$. It is known that the solutions to the D- and F-term equations intersect *generically* in a point [25]. That is, there are point-like field configurations which satisfy

$$D_a \;=\; F_i \;=\; 0 \quad \text{at } \phi^{(i)} \;=\; \langle\phi^{(i)}\rangle \;, \tag{2.2}$$

where, as usual,

$$D_a \;=\; \sum_i (\phi^{(i)})^* \, \mathsf{T}_a \, \phi^{(i)} \;, \tag{2.3a}$$

$$F^{(i)} \;=\; \frac{\partial \mathscr{W}}{\partial \phi^{(i)}} \;. \tag{2.3b}$$

The term 'point-like' means that there are no massless deformations of the vacuum (2.2). The reason why these vacua are point-like is easily understood: generically the F-term equations constitute as many gauge invariant constraints as there are gauge invariant variables. However, this also means that, at least generically,

$$\mathscr{W}|_{\phi^{(i)} \;=\; \langle\phi^{(i)}\rangle} \;\neq\; 0 \;. \tag{2.4}$$

If the fields attain VEVs $\langle\phi^{(i)}\rangle$ of the order of the fundamental scale, one hence expects to have too large a VEV for $\mathscr{W}$. One possible solution to the problem relies on approximate R symmetries [26], where one obtains a highly suppressed VEV of the superpotential. In what follows, we discuss an alternative: in settings with a residual R symmetry the above conclusion can be avoided as well.

2.2 Discrete R symmetries

Let us now discuss vacua with discrete R symmetries. To be specific, consider the order four symmetry $\mathbb{Z}_4^R$, under which the superpotential $\mathscr{W}$ has charge 2, such that

$$\mathscr{W} \;\stackrel{\zeta}{\to}\; -\mathscr{W} \tag{2.5}$$

under the $\mathbb{Z}_4^R$ generator ζ. Superspace coordinates transform as

$$\theta_\alpha \;\to\; \mathrm{i}\,\theta_\alpha \tag{2.6}$$

such that the F-term Lagrangean

$$\mathscr{L}_F = \int \mathrm{d}^2\theta\, \mathscr{W} + \text{h.c.} \tag{2.7}$$

is invariant. Chiral superfields will have R charges $0, 1, 2, 3$.[2] Both the fields of the type ψ_1 and ψ_3 with R charges 1 and 3, respectively, can acquire mass as the ψ_1^2 and ψ_3^2 terms have R charge $2 \mod 4$ and thus denote allowed superpotential terms.

The system of fields $\phi_0^{(i)}$ and $\psi_2^{(j)}$ with R charges 0 and 2, respectively, is more interesting. Consider first only one field ϕ_0 and one field ψ_2. The structure of the superpotential is

$$\mathscr{W} = \psi_2 \cdot f(\phi_0) + \mathcal{O}(\psi_2^3) \tag{2.8}$$

with some function f. The F-term for ϕ_0 vanishes trivially as long as $\mathbb{Z}_4^R$ is unbroken,

$$\frac{\partial \mathscr{W}}{\partial \phi_0} = \psi_2 \cdot f'(\phi_0) = 0 \,. \tag{2.9}$$

Note that due to the $\mathbb{Z}_4^R$ symmetry the superpotential vanishes in the vacuum. Thus it is sufficient to look at the global supersymmetry F-terms. On the other hand, the F-term constraint (at $\psi_2 = 0$)

$$\frac{\partial \mathscr{W}}{\partial \psi_2} = f(\phi_0) \stackrel{!}{=} 0 \tag{2.10}$$

will in general fix ϕ_0 at some non-trivial zero $\langle\phi_0\rangle$ of f. Indeed, there will be a supersymmetric mass term, which can be seen by expanding ϕ_0 around its VEV, i.e. inserting $\phi_0 = \langle\phi_0\rangle + \delta\phi_0$ into (2.8),

$$\mathscr{W} = f'(\langle\phi_0\rangle)\, \delta\phi_0\, \psi_2 + \mathcal{O}(\delta\phi_0^2, \psi_2^3) \,. \tag{2.11}$$

The supersymmetric mass $f'(\langle\phi_0\rangle)$ is generically different from 0.

Repeating this analysis for N $\phi_0^{(i)}$ and M $\psi_2^{(j)}$ fields reveals that the F-terms of the $\psi_2^{(j)}$ lead to M, in general independent, constraints on the $\phi_0^{(i)}$ VEVs. For $N = M$ we therefore expect point-like vacua with all directions fixed in a supersymmetric way.

To summarize, systems with a residual R symmetry ensure, unlike in the case without residual symmetries, that $\langle\mathscr{W}\rangle = 0$. However, in systems which exhibit a linearly realized $\mathbb{Z}_4^R$ somewhere in field space it may not be possible to find a supersymmetric vacuum that preserves $\mathbb{Z}_4^R$. In the case of a generic superpotential this happens if there are more, i.e. $M > N$, fields with R charge 2 than with 0. On the other hand, if there are more fields with R charge 0 than with 2, i.e. for $M < N$, one expects to have a Minkowski vacuum with $N - M$ flat directions. For $N = M$ one can have supersymmetric Minkowski vacua with all directions fixed in a supersymmetric way.

An important comment in this context concerns the moduli-dependence of couplings. As we have seen, in the case of discrete R-symmetries one might obtain more constraint (i.e. F-term) equations than R-even 'matter' fields. Specifically, in string vacua one should, however, carefully take into account all R-even fields, also the Kähler and complex structure moduli, T_i and U_j, on whose values the coupling strengths depend.

[2] A special role is played by the dilaton S, whose imaginary part $a = \operatorname{Im} S|_{\theta=0}$ shifts under $\mathbb{Z}_4^R$.

3 An explicit string-derived model

In order to render our discussion more specific, we base our analysis on a concrete model. We consider a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification with an additional freely acting $\mathbb{Z}_2$ of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string. Details of the model including shift vectors and Wilson lines can be found in appendix E.

In [27] a vacuum configuration of a very similar $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with matter parity and other desirable features was presented. However, the vacuum configuration discussed there has the unpleasant property that, at least generically, all Higgs fields attain large masses. In what follows we discuss how this can be avoided by identifying vacuum configurations with enhanced symmetries. In [19] another vacuum with the $\mathbb{Z}_4^R$ symmetry discussed in the introduction was found by using the methods presented in this paper. In both models the GUT symmetry is broken non-locally. This may be advantageous from the point of view of precision gauge unification [28]. It also avoids fractionally charged exotics, which appear in many other compactifications (cf. the discussion in [29]).

Labeling of states. We start our discussion with a comment on our notation. In a first step, we label the fields according to their $G_{\mathrm{SM}} \times [\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2)]_{\mathrm{hid}}$ quantum numbers. In particular, we denote the standard model representations with lepton/Higgs and d-quark quantum numbers as

$$L_i \ : \ (\mathbf{1},\mathbf{2})_{-1/2}\ , \tag{3.1a}$$

$$\bar{L}_i \ : \ (\mathbf{1},\mathbf{2})_{1/2}\ , \tag{3.1b}$$

$$D_i \ : \ (\mathbf{3},\mathbf{1})_{-1/3}\ , \tag{3.1c}$$

$$\bar{D}_i \ : \ (\overline{\mathbf{3}},\mathbf{1})_{1/3}\ . \tag{3.1d}$$

In the next step we identify $\mathbb{Z}_4^R$ such that the $\bar{L}_i/L_i$ decompose in lepton doublets ℓ_i with odd $\mathbb{Z}_4^R$ charges and Higgs candidates h_d/h_u with even $\mathbb{Z}_4^R$ charges etc. The details of labeling states are given in appendix E.

Searching for $\mathbb{Z}_4^R$. How can one obtain vacua with $\mathbb{Z}_4^R$ in practice? We found the following strategy most efficient:

1. In a first step we switch on a random sample of SM singlets in such a way that all unwanted gauge factors are spontaneously broken.
2. With these VEVs at hand, the original gauge and discrete symmetries at the orbifold point get broken to a discrete subgroup, which can be determined unambiguously with the methods described in [30]. Details of the automatization of these methods are explained in [31].
3. We only keep configurations in which there is a residual $\mathbb{Z}_4^R$ symmetry with precisely three generations of matter having R-charge 1; details of how to identify such configurations are given in appendix B.

4. From these configurations we select those exhibiting the following properties:
   - F- and D-flat;
   - all exotics decouple;
   - one pair of massless Higgs, i.e. μ term forbidden to all orders (at the perturbative level);
   - Yukawa couplings have full rank.

One of the main achievements of this study is a considerable simplification in the verification of the four items listed in step 4. In order to check D-flatness of a given configuration we use the Hilbert basis method, which is described in detail in appendix C. The other three properties can be verified by inspecting the remnant discrete symmetries only. In earlier studies [13–16] we had to explicitly identify couplings that are consistent with the string selection rules in order to show that all exotics decouple and the Yukawa couplings have full rank. In our new approach the remnant symmetries will tell us immediately whether an entry of a mass or Yukawa matrix will or will not appear. We have cross-checked this method extensively by explicitly computing the couplings between the charged and the VEV fields, and were always able to find a coupling which fills in an entry of a matrix, albeit sometimes at very high orders. Note, we assume that all couplings allowed by string selection rules appear in the superpotential.

VEV configuration. Following the above steps, we obtained a promising configuration in which the fields

$$\begin{aligned}\widetilde{\phi}^{(i)} &= \{\phi_1,\phi_2,\phi_3,\phi_4,\phi_5,\phi_6,\phi_7,\phi_8,\phi_9,\phi_{10},\phi_{11},\phi_{12},\phi_{13},\phi_{14},\\ &\quad x_1,x_2,x_3,x_4,x_5,\bar{x}_1,\bar{x}_3,\bar{x}_4,\bar{x}_5,y_3,y_4,y_5,y_6\}\end{aligned} \tag{3.2}$$

attain VEVs. The full quantum numbers of these fields are given in table E.2 in appendix E. In order to ensure D-flatness with respect to the hidden sector gauge factors, in a given basis not all components of the $x_i/\bar{x}_i$ and y_i attain VEVs. Details are given in equations (D.3) and (D.8) in appendix D.

Remnant discrete symmetries. By giving VEVs to the $\widetilde{\phi}^{(i)}$ fields in (3.2), we arrive at a vacuum in which, apart from G_{SM} and a 'hidden' SU(2), all gauge factors are spontaneously broken. The vacuum exhibits a $\mathbb{Z}_4^R$ symmetry, whereby the superpotential $\mathscr{W}$ has $\mathbb{Z}_4^R$ charge 2.

The $\mathbb{Z}_4^R$ charges of the matter fields are shown in table 3.1. The detailed origin of the $\mathbb{Z}_4^R$ symmetry is discussed later. Given these charges, we confirm by a straightforward field-theoretic calculation (cf. [24,32]) that $\mathbb{Z}_4^R$ appears indeed anomalous with universal $\mathrm{SU}(2)_\mathrm{L}-\mathrm{SU}(2)_\mathrm{L}-\mathbb{Z}_4^R$ and $\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_4^R$ anomalies (see [19] and appendix A.1). The statement that $\mathbb{Z}_4^R$ appears anomalous means, as we shall discuss in detail below, that the anomalies are cancelled by a Green-Schwarz (GS) mechanism. On the other hand, the $\mathbb{Z}_4^R$ has a, by the traditional criteria, non-anomalous $\mathbb{Z}_2^{\mathcal{M}}$ subgroup which is equivalent to matter parity [19].

(a) Quarks and leptons.

	q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$
$\mathbb{Z}_4^R$	1	1	1	1	1

(b) Higgs and exotics.

	h_1	h_2	h_3	h_4	h_5	h_6	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$	$\bar{h}_5$	$\bar{h}_6$	δ_1	δ_2	δ_3	$\bar{\delta}_1$	$\bar{\delta}_2$	$\bar{\delta}_3$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2	0	2	2	2	0	0

Table 3.1: $\mathbb{Z}_4^R$ charges of the (a) matter fields and (b) Higgs and exotics. The index i in (a) takes values $i = 1, 2, 3$.

***D*-flatness.** As already discussed, we cannot switch on the $\widetilde{\phi}^{(i)}$ fields at will; rather we have to show that there are vacuum configurations in which all these fields acquire VEVs. This requires to verify that the D- and F-term potentials vanish. With the Hilbert basis method (see appendix C) we could identify a complete set of D-flat directions composed of $\widetilde{\phi}^{(i)}$ fields. We compute the dimension of the D-flat moduli space using Singular [33] and the STRINGVACUA [34] package; the result is that there are 18 D-flat directions; the details of the computation are collected in appendix D.

***F*-term constraints.** Next we consider the F-term constraints. As discussed in section 2, the F-term conditions come from the fields with R-charge 2. We compute the number of independent conditions in appendix D. The result is that there are 23 independent conditions on $18 + 6 = 24$ D-flat directions, where we included the Kähler and complex structure moduli. We therefore expect to find supersymmetric vacuum configurations in which all the $\widetilde{\phi}^{(i)}$ acquire VEVs. In this configuration, almost all singlet fields, including the geometric moduli are fixed in a supersymmetric way. It will be interesting to compare this result to similar results found recently in the context of smooth heterotic compactifications [35]. We expect a significantly different, i.e. healthier, phenomenology than in the case in which a large number of singlets acquire mass only after supersymmetry breaking [36, 37]. Notice that there are two possible caveats. First, the analysis performed strictly applies only to superpotentials which are, apart from all the symmetries we discuss, generic. Second, it might happen that there are supersymmetric vacua, but they occur at large VEVs of some of the fields, i.e. in regions of field space where we no longer control our construction. Both issues will be addressed elsewhere.

Higgs vs. matter. The $\mathbb{Z}_2^{\mathcal{M}}$ subgroup of the $\mathbb{Z}_4^R$ symmetry allows us to discriminate between

- 3 lepton doublets, $\ell_i = \{L_4, L_6, L_7\}$,

- 3 d-type quarks, $\bar{d}_i = \{\bar{D}_1, \bar{D}_3, \bar{D}_4\}$,

on the one hand, and

- Higgs candidates, $h_i = \{L_1, L_2, L_3, L_5, L_8, L_9\}$ and $\bar{h}_i = \{\bar{L}_1, \bar{L}_2, \bar{L}_3, \bar{L}_4, \bar{L}_5, \bar{L}_6\}$,
- exotic triplets, $\delta_i = \{D_1, D_2, D_3\}$ and $\bar{\delta}_i = \{\bar{D}_2, \bar{D}_5, \bar{D}_6\}$

on the other hand.

Decoupling of exotics. With the charges in table 3.1 we can readily analyze the structure of the mass matrices. We crosscheck these structures by explicitly computing the couplings allowed by the string selection rules (cf. [27]). Note there is a caveat: our results are based on the assumption that all couplings that are allowed by the selection rules will appear with a non-vanishing coefficient. A $\widetilde{\phi}^n$ in the matrices represents a known polynomial of order n in the $\widetilde{\phi}$ fields which we have calculated using string selection rules. A zero entry in the matrices means that the corresponding coupling is not present in the perturbative superpotential. The $\bar{h}_i - h_j$ Higgs mass matrix is

$$\mathcal{M}_h \;=\; \begin{pmatrix} 0 & \phi_6 & 0 & \phi_4 & 0 & 0 \\ \phi_7 & 0 & \phi_2 & 0 & \phi_{13} & \phi_{14} \\ 0 & \phi_1 & 0 & \widetilde{\phi}^3 & 0 & 0 \\ 0 & \widetilde{\phi}^3 & 0 & \widetilde{\phi}^5 & 0 & 0 \\ \widetilde{\phi}^3 & 0 & \phi_{11} & 0 & \phi_8 & \widetilde{\phi}^3 \\ \widetilde{\phi}^3 & 0 & \phi_{12} & 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix} . \tag{3.3}$$

Here we omit coefficients, which depend on the three Kähler moduli T_i and complex structure moduli U_i. Clearly, this mass matrix has rank five, such that there is one massless Higgs pair

$$h_u \;=\; a_1\,\bar{h}_1 + a_2\,\bar{h}_3 + a_3\bar{h}_4\;, \tag{3.4a}$$
$$h_d \;=\; b_1\,h_1 + b_2\,h_3 + b_3\,h_5 + b_4\,h_6 \tag{3.4b}$$

with a_i and b_j denoting coefficients. The $\bar{\delta} - \delta$ mass matrix is

$$\mathcal{M}_\delta \;=\; \begin{pmatrix} \widetilde{\phi}^5 & 0 & 0 \\ 0 & \phi_8 & \widetilde{\phi}^3 \\ 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix} . \tag{3.5}$$

Hence, the matrix has full rank and all exotics decouple. Note that the block structure of $\mathcal{M}_\delta$ is not a coincidence but a consequence of the fact that δ_2/δ_3 and $\bar{\delta}_2/\bar{\delta}_3$ form D_4 doublets (see below). Altogether we see that all exotics with Higgs quantum numbers, and all but one pair of exotic triplets, decouple at the linear level in the $\widetilde{\phi}^{(i)}$ fields. This leads to the expectation that all but one pair of exotics get mass of the order of the GUT (or compactification) scale M_{GUT} while one pair of triplets might be somewhat lighter. We also note that the presence of colored states somewhat below M_{GUT} can give a better fit to MSSM gauge coupling unification (cf. [38]). However, a crucial property of the δ- and $\bar{\delta}$ triplets is that, due to the $\mathbb{Z}_4^R$ symmetry, they do not mediate dimension five proton decay.

Effective Yukawa couplings. The effective Yukawa couplings are defined by

$$\mathscr{W}_Y \;=\; \sum_{i=1,3,4}\left[(Y_u^{(i)})^{fg}\,q_f\,\bar{u}_g\,\bar{h}_i\right]+\sum_{i=1,3,5,6}\left[(Y_d^{(i)})^{fg}\,q_f\,\bar{d}_g\,h_i+(Y_e^{(i)})^{fg}\,\ell_f\,\bar{e}_g\,h_i\right]\;. \quad (3.6)$$

The Yukawa coupling structures are

$$Y_u^{(1)} \;=\; \begin{pmatrix} \widetilde{\phi}^2 & \widetilde{\phi}^4 & \widetilde{\phi}^6 \\ \widetilde{\phi}^4 & \widetilde{\phi}^2 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \end{pmatrix}\;,\quad Y_u^{(3)} \;=\; \begin{pmatrix} 1 & \widetilde{\phi}^6 & \widetilde{\phi}^4 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \\ \widetilde{\phi}^4 & \widetilde{\phi}^4 & \widetilde{\phi}^2 \end{pmatrix}\;, \quad (3.7a)$$

$$Y_e^{(5)} = (Y_d^{(5)})^T \;=\; \begin{pmatrix} \widetilde{\phi}^6 & \widetilde{\phi}^6 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \end{pmatrix}\;, \quad (3.7b)$$

$$Y_e^{(6)} = (Y_d^{(6)})^T \;=\; \begin{pmatrix} \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & \widetilde{\phi}^6 \\ 1 & \widetilde{\phi}^6 & \widetilde{\phi}^4 \end{pmatrix}\;. \quad (3.7c)$$

Y_d and Y_e coincide at tree-level, i.e. they exhibit SU(5) GUT relations, originating from the non-local GUT breaking due to the freely acting Wilson line. There are additional contributions to Y_u from couplings to $\bar{h}_4$ and to Y_e/Y_d from couplings to $h_{1,3}$ which can be neglected if the VEVs of the $\widetilde{\phi}^{(i)}$ fields are small.

Because of the localization of the matter fields, we expect the renormalizable (1,3) and (3,1) entries in $Y_e^{(6)}$ to be exponentially suppressed.

Gauge-top unification. The $(3,3)$ entry of Y_u is related to the gauge coupling. More precisely, in an orbifold GUT limit in which the first $\mathbb{Z}_2$ orbifold plane is larger than the other dimensions there is an SU(6) bulk gauge symmetry, and the ingredients of the top Yukawa coupling h_u (i.e. the fields $\bar{h}_{1,3,4}$), $\bar{u}_3$ and q_3 are bulk fields of this plane, i.e. hypermultiplets in the $N=2$ supersymmetric description. As discussed in [39], this implies that the top Yukawa coupling y_t and the unified gauge coupling g coincide at tree-level. Moreover, localization effects in the two larger dimensions [40] will lead to a slight reduction of the prediction of y_t at the high scale such that realistic top masses can be obtained.

***D*₄ flavor symmetry.** The block structure of the Yukawa matrices is not a coincidence but a consequence of a D_4 flavor symmetry [41], related to the vanishing Wilson line in the e_1 direction, $W_1=0$ (cf. e.g. [42]). The first two generations transform as a D_4 doublet, while the third generation is a D_4 singlet.

Neutrino masses. In our model we have 11 neutrinos, i.e. SM singlets whose charges are odd under $\mathbb{Z}_4^R$ meaning that they have odd $\mathbb{Z}_2^{\mathcal{M}}$ charge, where $\mathbb{Z}_2^{\mathcal{M}}$ is the matter parity subgroup of $\mathbb{Z}_4^R$. Their mass matrix has rank 11 at the perturbative level. The neutrino Yukawa coupling is a 3×11 matrix and has full rank. Hence the neutrino see-saw mechanism with many neutrinos [43] is at work.

Proton decay operators. The $\mathbb{Z}_4^R$ symmetry forbids all dimension four and five proton decay operators at the perturbative level [19]. In addition, the non-anomalous matter parity subgroup $\mathbb{Z}_2^{\mathcal{M}}$ forbids all dimension four operators also non-perturbatively. The dimension five operators like $q\,q\,q\,\ell$ are generated non-pertubatively, as we will discuss below.

Non-perturbative violation of $\mathbb{Z}_4^R$. Once we include the terms that are only forbidden by the $\mathbb{Z}_4^R$ symmetry, we obtain further couplings. An example for such an additional term is the dimension five proton decay operator,

$$\mathscr{W}_{np} \supset q_1\,q_1\,q_2\,\ell_1\,\mathrm{e}^{-a\,S}\,(x_4\bar{x}_5 + x_5\bar{x}_4)\left[\begin{pmatrix}\phi_{11}\\ \phi_{12}\end{pmatrix}\cdot\begin{pmatrix}\phi_{11}\\ \phi_{12}\end{pmatrix}\right]^3 \phi_4\,\phi_7^2\left[\begin{pmatrix}\phi_9\\ \phi_{10}\end{pmatrix}\cdot\begin{pmatrix}\phi_9\\ \phi_{10}\end{pmatrix}\right] \tag{3.8}$$

where we suppressed coefficients. The bracket structure between the ϕ_{11}/ϕ_{12} and ϕ_9/ϕ_{10} is a consequence of the non-Abelian D_4 symmetry, where these fields transform as a doublet. The dot '$\cdot$' indicates the standard scalar product. Note that there are invariants with more than two D_4 charged fields which cannot be written in terms of a scalar product. Further, S is the dilaton and the coefficient $a = 8\pi^2$ in $\mathrm{e}^{-a\,S}$ is such that $\mathrm{e}^{-a\,S}$ has positive anomalous charge with respect to the normalized generator of the 'anomalous' U(1). This generator is chosen such that it is the gauge embedding of the anomalous space group element[3] (cf. equation (E.5)),

$$\mathsf{t}_{\mathrm{anom}} = W_3 + \mathrm{E}_8\times\mathrm{E}_8 \text{ lattice vectors}\,. \tag{3.9}$$

The discrete Green-Schwarz mechanism is discussed in detail in [44].

Solution to the μ problem. The $\mathbb{Z}_4^R$ anomaly has important consequences for the MSSM μ problem. The μ term is forbidden perturbatively by $\mathbb{Z}_4^R$, however, it appears at the non-perturbative level. Further, this model shares with the mini-landscape models the property that any allowed superpotential term can serve as an effective μ term (cf. the discussion in [26]). This fact can be seen from higher-dimensional gauge invariance [45]. Therefore, the (non-perturbative) μ term is of the order of the gravitino mass,

$$\mu \sim \langle\mathscr{W}\rangle \sim m_{3/2} \tag{3.10}$$

in Planck units. If some 'hidden' sector dynamics induces a non-trivial $\langle\mathscr{W}\rangle$, the μ problem is solved.

In our model, we have only a 'toy' hidden sector with an unbroken SU(2) gauge group and one pair of massless doublets whose mass term is prohibited by $\mathbb{Z}_4^R$. This sector has the structure discussed by Affleck, Dine and Seiberg (ADS) [46]. We find that the ADS superpotential is $\mathbb{Z}_4^R$ covariant. However, the hidden gauge group is probably too small for generating a realistic scale of supersymmetry breakdown. Yet there are alternative ways, such as the one described in [26], for generating a hierarchically small $\langle\mathscr{W}\rangle$.

[3]See [24] for the discussion in a more general context. Note that we can always bring the anomalous space group element to the form $(\theta^k\,\omega^\ell, 0)$ by redefining the model input appropriately. This amounts to a redefinition of the 'origin' of the orbifold.

Origin of $\mathbb{Z}_4^R$. In the orbifold CFT description the $\mathbb{Z}_4^R$ originates from the so-called H-momentum selection rules [47] (see also [14, 48]). These selection rules appear as discrete R symmetries in the effective field theory description of the model. We would like to stress that in large parts of the literature the order of these symmetries was given in an unfortunate way. This criticism also applies to the papers by some of the authors of this study. For instance, the $\mathbb{Z}_2$ orbifold plane was said to lead to a $\mathbb{Z}_2^R$ symmetry, but it turned out that there are states with half-integer charges. We find it more appropriate to call this symmetry $\mathbb{Z}_4^R$, and to deal with integer charges only. In our model we have three $\mathbb{Z}_4^R$ symmetries at the orbifold point, stemming from the three $\mathbb{Z}_2$ orbifold planes.

H-momentum corresponds to angular momentum in the compact space; therefore the discrete R symmetries can be thought of as discrete remnants of the Lorentz symmetry of internal dimensions. That is to say that the orbifold compactification breaks the Lorentz group of the tangent space to a discrete subgroup. In this study we content ourselves with the understanding that these symmetries appear in the CFT governing the correlators to which we match the couplings of our effective field theory. The precise geometric interpretation of this symmetry in field theory will be discussed elsewhere.

The actual $\mathbb{Z}_4^R$ charges of $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2)]_{\mathrm{hid}}$ invariant expressions in this model are given by

$$q_{\mathbb{Z}_4^R} \;=\; q_X + R_2 + 2n_3 \;, \tag{3.11}$$

where q_X is the U(1) charge generated by

$$\mathtt{t}_X \;=\; (4, 0, 10, -10, -10, -10, -10, -10)\;(-10, 0, 5, 5, -5, 15, -10, 0)\;, \tag{3.12}$$

R_2 denotes the R charge with respect to the second orbifold plane and n_3 is the localization quantum number in the third torus. The relevant quantum numbers are given in table E.2. The expression (3.11) for $q_{\mathbb{Z}_4^R}$ is not unique, there are 17 linear combinations of U(1) charges and discrete quantum numbers which can be used to rewrite the formula without changing the $\mathbb{Z}_4^R$ charges. Also the U(1) factors contained in $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2)]_{\mathrm{hid}}$ can be used to redefine $\mathtt{t}_X$. We refrain from spelling this out as we find it more convenient to work with invariant monomials (cf. the discussion in appendix D). It is straightforward to see that all monomials we switch on have R charge 0.

4 Summary

We have re-emphasized the important role of discrete symmetries in string model building. As an application, we discussed an explicit string model which exhibits MSSM vacua with a $\mathbb{Z}_4^R$ symmetry, which has recently been shown to be the unique symmetry for the MSSM that forbids the μ term at the perturbative level, allows Yukawa couplings and neutrino masses, and commutes with SO(10). This $\mathbb{Z}_4^R$ has a couple of appealing features. First, the μ term and dangerous dimension five proton decay operators are forbidden at the perturbative level and appear only through (highly suppressed) non-perturbative effects. Second, at the perturbative level, the expectation value of the

superpotential is zero; a non-trivial expectation value is generated by non-perturbative effects. These two points imply that μ is of the order of the gravitino mass $m_{3/2}$, which is set by the expectation value of the superpotential (in Planck units).

The model is a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string. We discussed how to search for field configurations which preserve $\mathbb{Z}_4^R$ and how to find supersymmetric vacua within such configurations. The Hilbert basis method allowed us to construct a basis for all gauge invariant holomorphic monomials, and therefore to survey the possibilities of satisfying the D-term constraints. As we have seen, in the case of residual R symmetries it may in principle happen that the F-term equations overconstrain the system. We have explicitly verified that this is not the case in our model, i.e. there are supersymmetric vacua with the exact MSSM spectrum and a residual $\mathbb{Z}_4^R$ symmetry. Let us highlight the features of the model:

- exact MSSM spectrum, i.e. no exotics;
- almost all singlet fields/moduli are fixed in a supersymmetric way;
- non-local GUT breaking, i.e. the model is consistent with MSSM precision gauge unification;
- dimension four proton decay operators are completely absent as $\mathbb{Z}_4^R$ contains the usual matter parity as a subgroup;
- dimension five proton decay operators only appear at the non-perturbative level and are completely harmless;
- the gauge and top-Yukawa couplings coincide at tree level;
- see-saw suppressed neutrino masses;
- μ is related to the vacuum expectation value of the superpotential and therefore of the order of the gravitino mass;
- there is an SU(5) GUT relation between the τ and bottom masses.

There are also two drawbacks: first, there are also SU(5) relations for the light generations and second the hidden sector gauge group is only SU(2) and therefore probably too small for explaining an appropriate scale of dynamical supersymmetry breaking.

Although we have obtained a quite promising string vacuum, the main focus of this paper was on developing new methods rather than working out the phenomenology of a model. We have discussed in detail how to determine the residual discrete symmetries of a given VEV configuration. As a consequence, we could immediately understand the features of such a configuration. For instance, in earlier studies [13–16] we had to explicitly identify couplings that are consistent with the string selection rules in order to show that all exotics decouple and the Yukawa couplings have full rank. This is a very time-consuming task in practice. With the new methods we could obtain this information by just looking at the remnant symmetries. We have performed extensive

cross-checks in order to show that both methods yield the same results. We have also shown how to search for vacua with a given symmetry. Further, we presented the Hibert basis method which allows us to survey all D-flat directions comprised of a selected set of fields in very short time. It will be interesting to apply, and to extend, our methods to other examples.

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A Discrete anomalies

A.1 Discrete anomaly calculation

We calculate the anomaly of the $\mathbb{Z}_4^R$ symmetry of the configuration discussed in section 3. The fermions of a superfield with $\mathbb{Z}_4^R$ charge q have $\mathbb{Z}_4^R$ charge $q-1$ because we work in a convention where the superpotential carries charge two. Only massless states contribute to the anomaly (cf. [24]). We can hence limit ourselves to the MSSM field content. Since all matter fields carry $\mathbb{Z}_4^R$ charge one, the corresponding fermion is uncharged and does not contribute to the anomaly. Thus, we end up with the following contributions:

$\mathrm{SU}(3)_C$				$\mathrm{SU}(2)_\mathrm{L}$			
				$h_{u,d}$	$2\cdot\frac{1}{2}\cdot 3$	$=$	3
gauginos	$c_2(\mathbf{8})$	$=$	3	gauginos	$c_2(\mathbf{3})$	$=$	2
			3				5

The factor 1/2 in the Higgs contributions is the Dynkin index. The anomaly condition is that the sum over all charges be equal mod 2. The total contribution in both cases is odd. That is, both symmetries appear anomalous, and the anomaly is, in particular, universal, as required for the Green-Schwarz mechanism to work.

A.2 Anomaly Mixing

In our model we have two symmetries which appear anomalous. On the one hand, we have the anomalous $\mathrm{U}(1)_\mathrm{anom}$. On the other hand, the space group selection rule $\mathbb{Z}_2^{n_3}$ for n_3 (cf. [27]), corresponding to the anomalous space group element in equation (E.5), is

anomalous. We wish to answer the question if it is possible to rotate the $\mathbb{Z}_2^{n_3}$ anomaly completely into $\mathrm{U}(1)_{\mathrm{anom}}$.

Consider a setting with $\mathrm{U}(1) \times \mathbb{Z}_N$ symmetry which appear anomalous. We will denote the U(1) charge by $Q^{(i)}$ and the $\mathbb{Z}_N$ charges by $q^{(i)}$, and we will assume all charges to be integers. Suppose we have a gauge group G. The anomaly coefficients read

$$G - G - \mathrm{U}(1) \ : \quad \sum_f Q^{(f)}\,\ell(\boldsymbol{r}^{(f)}) \ = \ A\,, \tag{A.1a}$$

$$G - G - \mathbb{Z}_N \ : \quad \sum_f q^{(f)}\,\ell(\boldsymbol{r}^{(f)}) \ = \ B\,, \tag{A.1b}$$

where the sums run over the irreducible representations of G and $\ell(\boldsymbol{r})$ is the Dynkin index of the representation $\boldsymbol{r}$. Specifically, $\ell = 1/2$ for the fundamental representation of $\mathrm{SU}(N)$.

We can redefine the $\mathbb{Z}_N$ charges by shifting them by integer multiples of the U(1) charges. That is, we can define new $\mathbb{Z}_N$ charges $q'^{(i)} = q^{(i)} + n\,Q^{(i)}$. Then the new $G - G - \mathbb{Z}_N$ anomaly coefficient is given by

$$\sum_f q'^{(f)}\ell(\boldsymbol{r}^{(f)}) \ = \ \sum_f \left(q^{(f)} + n\,Q^{(f)}\right)\,\ell(\boldsymbol{r}^{(f)}) \ = \ B + n\,A \tag{A.2}$$

with $n \in \mathbb{Z}$. Anomaly freedom requires [24,32]

$$\sum_f q'^{(f)}\ell(\boldsymbol{r}^{(f)}) \ = \ 0 \quad \mathrm{mod}\ \eta \quad \text{where} \quad \eta \ = \ \begin{cases} N, & N\ \text{odd} \\ \frac{N}{2}, & N\ \text{even} \end{cases} . \tag{A.3}$$

Hence, the $\mathbb{Z}_N$ can be made anomaly-free if there is a solution to

$$B + n\,A \ = \ 0 \quad \mathrm{mod}\ \eta \tag{A.4}$$

for $n \in \mathbb{Z}$.

The anomaly coefficient for the anomalous $\mathrm{U}(1)_{\mathrm{anom}}$ in our model is given by

$$\mathrm{SU}(3)_C - \mathrm{SU}(3)_C - \mathrm{U}(1)_{\mathrm{anom}} \ : \quad A \ = \ 15 \tag{A.5}$$

for the SM gauge group $\mathrm{SU}(3)_C$. The $\mathrm{U}(1)_{\mathrm{anom}}$ charges are normalized to integers. The anomaly coefficients are the same for $\mathrm{SU}(2)_{\mathrm{L}}$ and for the gauge group factors of the hidden sector because of the Green-Schwarz mechanism. In addition, the $\mathbb{Z}_2^{n_3}$ anomaly coefficient turns out to be

$$\mathrm{SU}(3)_C - \mathrm{SU}(3)_C - \mathbb{Z}_2^{n_3} \ : \quad B \ = \ \frac{1}{2}\,. \tag{A.6}$$

The equation

$$B + n\,A \ = \ \frac{1}{2} + 15\,n \ = \ 0 \quad \mathrm{mod}\ 1 \tag{A.7}$$

has no solution for $n \in \mathbb{Z}$ and therefore the anomaly of $\mathbb{Z}_2^{n_3}$ cannot be removed. Altogether we have demonstrated that there are (at least) two independent anomalies, i.e. the imaginary part of the dilaton shifts both under $\mathrm{U}(1)_{\mathrm{anom}}$ and $\mathbb{Z}_2^{n_3}$ transformations and there is no way of rotating the $\mathbb{Z}_2^{n_3}$ anomaly into $\mathrm{U}(1)_{\mathrm{anom}}$.

B Identifying the $\mathbb{Z}_4^R$ symmetry

In section 3 we explained our strategy for constructing promising vacuum configurations. A crucial point is the identification of vacua that exhibit a $\mathbb{Z}_4^R$ symmetry, with charges given in table 3.1. In this appendix, we will give some details of how this identification is done.

In a particular vacuum, our models exhibit Abelian discrete symmetries which are calculated by the methods described in [30]. A finite Abelian group can always be written as a direct product $G = H_{p_1} \times \ldots \times H_{p_n}$ where the p_i are pairwise distinct primes and $H_p = \mathbb{Z}_{p^{e_1}} \times \ldots \times \mathbb{Z}_{p^{e_m}}$ with $e_1 \leq \cdots \leq e_m$ positive integers. In a first step, we have to make sure that G has a $\mathbb{Z}_4$ subgroup. This can be done by looking at the subgroup H_2 of G.

In the next step, we want to know whether the $\mathbb{Z}_4$ is of R or non-R type. In order to answer this question, we only have to look at the transformation of the superpotential under the $\mathbb{Z}_4$; thus one can unambiguously see if the $\mathbb{Z}_4$ is R or non-R.

If the considered vacuum exhibits a $\mathbb{Z}_4^R$ symmetry, we have to check that the charges of the matter fields match the ones in table 3.1. A technical problem appears if H_2 consists of more than one factor, i.e. if there is an additional $\mathbb{Z}_{2^n}$ symmetry where n is a positive integer. In this case there are many equivalent charge assignments some of which will make the $\mathbb{Z}_4^R$ obvious while others will conceal its existence. This freedom corresponds to the automorphisms of G [49] whose number can be large, e.g. $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_4$ has 1536 automorphisms. An important fact is that the automorphism group factorizes in the way we have written G, i.e. $\mathrm{Aut}(H_{p_1} \times H_{p_2}) \cong \mathrm{Aut}(H_{p_1}) \times \mathrm{Aut}(H_{p_2})$. Thus, we only need to look at H_2.

The automorphisms can be represented by certain matrices acting on charge vectors [49]. To see whether the $\mathbb{Z}_4^R$ is present in a vacuum, we scan over all possible charge assignments of H_2 and look for a $\mathbb{Z}_4$ subgroup under which all SM matter fields have charge 1. The other states can be even or odd, as long as they are vector-like and can be decoupled.

To illustrate this procedure, let us look at a simple example in table B.1. The two charge assignments are equivalent. While in (a) it is not obvious that there is a $\mathbb{Z}_4$ subgroup under which all ψ have charge 1, in the second charge assignment in (b) all fields have charge 1 with respect to the second $\mathbb{Z}_4$ subgroup.

C Hilbert bases and D-flatness

C.1 General discussion

In this appendix, a simple method is described that allows us to analyze D-flatness.

It is well known that D-flat directions correspond to holomorphic gauge invariant monomials [50, 51]. We will present now a method to compute all holomorphic gauge invariant monomials. Let us look at a theory with gauge group $\mathrm{U}(1)^n$. As can be easily seen, a monomial $\phi_1^{n_1}\phi_2^{n_2}\ldots\phi_k^{n_k}$ is D-flat under the j^{th} U(1) factor if (cf. e.g. [16,

(a)

	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
ψ_1	1	0	3
ψ_2	1	3	2
ψ_3	0	0	3

(b)

	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
ψ_1	0	1	2
ψ_2	1	1	3
ψ_3	1	1	0

Table B.1: An example for a hidden $\mathbb{Z}_4$ symmetry under which all fields have charge 1. The two charge assignments are equivalent.

appendix B, equation (B3)])

$$\sum_i q_i^{(j)} n_i \;=\; 0 \tag{C.1}$$

where $q_i^{(j)}$ is the charge of the field ϕ_i under the j^{th} U(1). The index i runs from 1 to the number of fields k in the monomial and the index j over the number of different U(1) factors. We can rewrite this as a matrix equation,

$$\begin{pmatrix} q_1^{(1)} & \dots & q_k^{(1)} \\ \vdots & & \vdots \\ q_1^{(n)} & \dots & q_k^{(n)} \end{pmatrix} \cdot \begin{pmatrix} n_1 \\ \vdots \\ n_k \end{pmatrix} \;=\; 0\;, \tag{C.2}$$

where n_i counts how often a field occurs in the monomial. The charges can always be scaled to become integers whereas the field multiplicity n_i has to be a non-negative integer. Thus, the condition to have a D-flat monomial is

$$Q\cdot x \;=\; 0\;,\quad Q\in\mathbb{Z}_{n\times k}\;,\quad x\in\mathbb{N}^k\;. \tag{C.3}$$

This is a system of homogeneous linear Diophantine equation over non-negative integers. Such equations can be solved completely by constructing the corresponding Hilbert basis (see for example [52]). A Hilbert basis $\mathcal{H}(Q)$ is a complete set of all minimal solutions to equation (C.3). A solution is called minimal if it is non-trivial and there exists no smaller solution. Here 'smaller' means that, given a solution x, there is no other solution $y\neq x$ with $y_i\leq x_i$ for all $i=1\dots k$. Given $\mathcal{H}(Q)$ we can construct all non-negative solutions to equation (C.3) by forming linear combinations of the basis solutions with non-negative integer coefficients. The Hilbert basis $\mathcal{H}(Q)$ is therefore a basis for all D-flat monomials. In practice the Hilbert basis for a given matrix Q can be computed with the help of computer algebra packages like [53]. Such packages can be applied even for rather large matrices.

Let us look at an example from [25] with four fields. The charges of the fields are summarized in the charge matrix

$$Q \;=\; \begin{pmatrix} 2 & -2 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix}\;. \tag{C.4}$$

The Hilbert basis $\mathcal{H}(Q)$ is given by the three vectors

$$x = \begin{pmatrix}1\\0\\0\\2\end{pmatrix}, \quad y = \begin{pmatrix}1\\2\\2\\0\end{pmatrix}, \quad z = \begin{pmatrix}1\\1\\1\\1\end{pmatrix} \qquad x, y, z \in \mathcal{H}(Q) . \tag{C.5}$$

All holomorphic gauge invariant monomials $\phi_1^{n_1}\phi_2^{n_2}\phi_3^{n_3}\phi_4^{n_4}$ can be characterized by four-vectors $w = (n_1, n_2, n_3, n_4)^T$, which are given by $w = \alpha\, x + \beta\, y + \gamma\, z$ with $\alpha, \beta, \gamma \in \mathbb{N}$. We recognize an important property of the Hilbert basis: while the dimension of D-flat directions is 2, i.e. the number of fields minus the number of independent D-term constraints, the length of the Hilbert basis is larger, namely 3. There is one relation between the Hilbert basis elements,

$$x + y = 2z . \tag{C.6}$$

This is the price one has to pay for being able to express any D-flat direction as an integer linear combination of basis monomials.

C.2 Hilbert basis for the vacuum discussed in section 3

With the Hilbert basis method we could identify a complete set of D-flat directions composed of $\widetilde{\phi}$ fields in equation (3.2). As discussed, the key feature of this Hilbert basis is that any gauge invariant holomorphic monomial can be expressed as product of the basic monomials. We obtain 6184 monomials. An example of a monomial, which has negative charge under the anomalous U(1), is given in equation (D.9).

D Details of the supersymmetric vacuum configuration

D.1 *D*-flatness

Hidden SU(2) breaking. We first look at SU(2). We find that we can switch on $y_3, \dots, y_6$. This leads to the 6 SU(2) invariant monomials

$$\{\mathscr{M}^{(i)}\}_{\mathrm{SU}(2)} = \{y_3y_4, y_3y_5, y_3y_6, y_4y_5, y_4y_6, y_5y_6\} . \tag{D.1}$$

There is one relation between the monomials,

$$(y_3y_4)\,(y_5y_6) - (y_3y_5)\,(y_4y_6) + (y_3y_6)\,(y_4y_5) = 0 , \tag{D.2}$$

such that the number of flat directions is 5, which is consistent with 8 components being switched on whereby 3 directions get eaten by the SU(2) gauge multiplet. The monomial y_1y_2, which also has R charge 0, vanishes. A possible way to have the above monomials non-vanishing is to set

$$y_1^{[1]} = y_1^{[2]} = y_2^{[1]} = y_2^{[2]} = y_3^{[1]} = y_4^{[2]} = 0 \tag{D.3a}$$

and to have, correspondingly,

$$y_3^{[2]},\ y_4^{[1]},\ y_5^{[1,2]},\ y_6^{[1,2]}\ \neq\ 0\ . \tag{D.3b}$$

Hidden SU(3) breaking. There are 16 SU(3) invariant composites of hidden SU(3) **3**- and $\overline{\mathbf{3}}$-plets with R charge 0, and can, from this perspective, acquire a VEV, namely

$$\begin{aligned}\{\mathscr{M}_0^{(i)}\}_{\mathrm{SU}(3)} &= \{x_1\overline{x}_1, x_2\overline{x}_3, x_2\overline{x}_4, x_2\overline{x}_5, \overline{x}_3x_4, \overline{x}_3x_5, x_4\overline{x}_4, x_4\overline{x}_5, x_5\overline{x}_4, x_5\overline{x}_5,\\ &\quad x_1x_2x_3, x_1x_3x_4, x_1x_3x_5, \overline{x}_2\overline{x}_3\overline{x}_4, \overline{x}_2\overline{x}_3\overline{x}_5, \overline{x}_2\overline{x}_4\overline{x}_5\}\ .\end{aligned} \tag{D.4}$$

There are different branches of SU(3) flat directions. In what follows we discuss one particular of them.

In this branch there are 13 composites with non-zero VEV,

$$\begin{aligned}\{\mathscr{M}^{(i)}\}_{\mathrm{SU}(3)} &= \{x_1\overline{x}_1, x_2\overline{x}_3, x_2\overline{x}_4, x_2\overline{x}_5, \overline{x}_3x_4, \overline{x}_3x_5, x_4\overline{x}_4, x_4\overline{x}_5, x_5\overline{x}_4, x_5\overline{x}_5,\\ &\quad x_1x_2x_3, x_1x_3x_4, x_1x_3x_5\}\ ,\end{aligned} \tag{D.5}$$

while the other 3 monomials vanish,

$$(\overline{x}_2\overline{x}_4\overline{x}_5)\ =\ (\overline{x}_2\overline{x}_3\overline{x}_5)\ =\ (\overline{x}_2\overline{x}_3\overline{x}_4)\ =\ 0\ . \tag{D.6a}$$

Assuming that the above $\{\mathscr{M}^{(i)}\}_{\mathrm{SU}(3)}$ VEVs do not vanish, we arrive at the relations

$$(\overline{x}_3x_4) = \frac{(\overline{x}_3x_5)\,(x_1x_3x_4)}{(x_1x_3x_5)}\ , \qquad (x_2\overline{x}_3) = -\frac{(\overline{x}_3x_5)\,(x_1x_2x_3)}{(x_1x_3x_5)}\ , \tag{D.6b}$$

$$(x_2\overline{x}_5) = -\frac{(x_1x_2x_3)\,(x_4\overline{x}_5)}{(x_1x_3x_4)}\ , \qquad (x_5\overline{x}_5) = \frac{(x_1x_3x_5)\,(x_4\overline{x}_5)}{(x_1x_3x_4)}\ , \tag{D.6c}$$

$$(x_2\overline{x}_4) = -\frac{(x_1x_2x_3)\,(x_4\overline{x}_4)}{(x_1x_3x_4)}\ , \qquad (x_5\overline{x}_4) = \frac{(x_1x_3x_5)\,(x_4\overline{x}_4)}{(x_1x_3x_4)}\ . \tag{D.6d}$$

This leaves us with 7 independent SU(3) monomials, a possible choice is given by

$$\{\overline{x}_3x_5, x_1\overline{x}_1, x_4\overline{x}_4, x_4\overline{x}_5, x_1x_2x_3, x_1x_3x_4, x_1x_3x_5\}\ . \tag{D.7}$$

So we see explicitly that this branch of D-flat directions has dimension 7. This is in agreement with the result obtained with the STRINGVACUA package [34].

We can satisfy the constraints by setting (in an appropriate gauge) various components to zero. The only non-vanishing components are

$$x_1^{[3]},\ x_2^{[2]},\ x_3^{[1]},\ x_4^{[2]},\ x_5^{[2]},\ \overline{x}_1^{[3]},\ \overline{x}_3^{[2]},\ \overline{x}_4^{[2]},\ \overline{x}_5^{[2]}\ \neq\ 0\ . \tag{D.8}$$

Abelian singlets. We can switch on the Abelian singlets

$$\{\mathscr{M}_0^{(i)}\}_{\mathrm{singlet}}\ =\ \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}, \phi_{11}, \phi_{12}, \phi_{13}, \phi_{14}\}\ .$$

Since there are 8 U(1) factors that get broken, there are 8 additional D-term constraints.

Cancellation of the FI term. With the Hilbert basis method we were able to compute all gauge invariant monomials carrying negative charge with repect to $\mathrm{U}(1)_{\mathrm{anom}}$. An example is

$$\mathscr{M}_{\mathrm{FI}} \;=\; \phi_{11}^4\,\phi_4\,\phi_7^2\,\phi_8\,\phi_9^2\;. \tag{D.9}$$

D.2 $\boldsymbol{F}$-flatness

Remnant $\mathbb{Z}_4^R$ symmetry. Switching on the above fields breaks the gauge, R and other discrete symmetries down to $G_{\mathrm{SM}} \times \mathbb{Z}_4^R \times [\mathrm{SU}(2)]$. We decompose the moduli space in SU(3) and SU(2) composites and basic fields $\mathscr{M}_r^{(m)}$ where r denotes the $\mathbb{Z}_4^R$ charge of the corresponding objects. A prominent role will be played by the singlet fields with R charge 2, which are given by

$$\begin{aligned}
\{\mathscr{M}_2^{(i)}\}_{\mathrm{SU}(3)} \;&=\; \{x_1\overline{x}_3, x_1\overline{x}_4, x_1\overline{x}_5, \overline{x}_1x_2, \overline{x}_1x_4, \overline{x}_1x_5, \overline{x}_2x_3,\\
&\quad\;\; x_2x_3x_4, x_2x_3x_5, x_3x_4x_5, \overline{x}_1\overline{x}_2\overline{x}_3, \overline{x}_1\overline{x}_2\overline{x}_4, \overline{x}_1\overline{x}_2\overline{x}_5\}\;,\\
\{\mathscr{M}_2^{(i)}\}_{\mathrm{SU}(2)} \;&=\; \{y_1y_3, y_1y_4, y_1y_5, y_1y_6, y_2y_3, y_2y_4, y_2y_5, y_2y_6\}\;,\\
\{\mathscr{M}_2^{(i)}\}_{\mathrm{singlet}} \;&=\; \{\overline{\phi}_1, \overline{\phi}_2, \overline{\phi}_3, \overline{\phi}_4, \overline{\phi}_5, \overline{\phi}_6, \overline{\phi}_7, \overline{\phi}_8, \overline{\phi}_9, \overline{\phi}_{10}, \overline{\phi}_{11}, \overline{\phi}_{12}\}\;.
\end{aligned} \tag{D.10}$$

***F*-term constraints.** One can use the above monomials for counting the independent F-term constraints. As discussed in section 2, the superpotential will be of the form

$$\mathscr{W} \;=\; \sum_m \mathscr{M}_2^{(m)}\cdot f_2^{(m)}\left(\mathscr{M}_0^{(1)},\dots\right) + \dots\;, \tag{D.11}$$

where the omission contains only terms which are at least quadratic in $\mathscr{M}_{\geq 1}^{(m)}$, and the $f_2^{(m)}$ are some functions of the monomials with R charge 0. The potentially non-trivial F-terms are then

$$\left.\frac{\partial\mathscr{W}}{\partial\phi_i}\right|_{\phi_i\,=\,\langle\phi_i\rangle} \;=\; \left.\sum_m \frac{\partial\mathscr{M}_2^{(m)}}{\partial\phi_i}\cdot f_2^{(m)}\left(\mathscr{M}_0^{(1)},\dots\right)\right|_{\phi_i\,=\,\langle\phi_i\rangle} \tag{D.12}$$

as we look at vacua with unbroken $\mathbb{Z}_4^R$, i.e. $\mathscr{M}_{\geq 1}^{(m)} = 0$. For $\mathscr{M}_2^{(m)} \in \{\mathscr{M}_2^{(i)}\}_{\mathrm{singlet}}$ each equation gives a non-trivial constraint on the $\mathscr{M}_0^{(j)}$. The number of independent constraints is given by the rank of the matrix

$$\mathscr{N} \;=\; (\mathscr{N}_{ij}) \;=\; \frac{\partial\mathscr{M}_2^{(i)}}{\partial\phi^{(j)}}\;, \tag{D.13}$$

where the $\phi^{(j)}$ comprise all component fields appearing in monomials, evaluated at the vacuum.

We evaluated the rank of the $\mathscr{N}$ matrix for the SU(3) and SU(2) monomials in the vacuum defined by (D.3) and (D.8). The result is that there are 7 independent F-term

constraints in the SU(3) case and 4 in the SU(2) case. Adding the constraints from the non-Abelian singlets with R charge 2, we therefore obtain $7+4+12=23$ F-term conditions. At this point, the supersymmetry conditions seem to over-constrain the system, as the number of D-flat directions is $7+5+14-8=18$. Note, however, that there are 6 additional degrees of freedom which we have not discussed yet: the T_i- and U_i-moduli of our $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold. The functions $f_2^{(m)}$ will also depend on these fields, which obviously have R charge 0 (cf. the discussion in [45]). Using these additional degrees of freedom we will *generically* be able to satisfy the constraints, and *generically* there will be only one flat direction in the moduli space formed out of the standard model singlet degrees of freedom![4]

There are also SU(3) invariant composites with odd $\mathbb{Z}_4^R$ charge. The superpotential will contain terms of the form

$$\mathscr{W} \supset \sum_{m,n}\mathscr{M}_1^{(m)}\cdot\mathscr{M}_1^{(n)}\cdot f_1^{(m,n)}\left(\mathscr{M}_0^{(1)},\dots\right) \tag{D.14}$$

and analogous terms for the $\mathscr{M}_3^{(m)}$. This will then lead to non-trivial mass terms for the vanishing SU(3) triplets and anti-triplets. Analogous statements hold for the other fields with odd R charges.

In summary, we expect that the vacuum discussed here is such that supersymmetry conditions can be satisfied. Moreover, we find that all but one of the fields are fixed by the D- and F-term constraints. Unlike in the case without a residual $\mathbb{Z}_4^R$ symmetry, due to the $\mathbb{Z}_4^R$ the superpotential expectation value is guaranteed to vanish at the perturbative level.

E Details of the model

The orbifold model is defined by a torus lattice that is spanned by six orthogonal vectors e_α, $\alpha=1,\dots,6$, the $\mathbb{Z}_2\times\mathbb{Z}_2$ twist vectors $v_1=(0,1/2,-1/2)$ and $v_2=(-1/2,0,1/2)$, and the associated shifts

$$V_1 = \left(-\frac{1}{2},-\frac{1}{2},0,0,0,0,0,0\right)(0,0,0,0,0,0,0,0)\ , \tag{E.1a}$$

$$V_2 = \left(0,\frac{1}{2},-\frac{1}{2},0,0,0,0,0\right)(0,0,0,0,0,0,0,0)\ , \tag{E.1b}$$

[4] Of course, there is the hidden SU(2) sector which contains further massless degrees of freedom. We kept this SU(2) unbroken on purpose as it may serve as a toy hidden sector for dynamical supersymmetry breakdown.

and the six discrete Wilson lines

$$
\begin{aligned}
W_1 &= \left(0^8\right)\left(0^8\right) , &\text{(E.2a)}\\
W_3 &= \left(\frac{3}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)\left(0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},1,1\right) , &\text{(E.2b)}\\
W_5 &= \left(-\frac{7}{4},\frac{7}{4},-\frac{1}{4},-\frac{3}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},-\frac{3}{4}\right)\left(-\frac{3}{4},\frac{5}{4},-\frac{5}{4},-\frac{5}{4},\frac{1}{4},\frac{1}{4},-\frac{3}{4},\frac{5}{4}\right) , &\text{(E.2c)}\\
W_6 &= \left(\frac{3}{2},\frac{1}{2},-\frac{3}{2},-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\frac{1}{2},\frac{1}{2}\right)\left(-\frac{3}{2},-\frac{1}{2},-\frac{1}{2},\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{3}{2},\frac{3}{2}\right) &\text{(E.2d)}\\
W_2 &= W_4 \;=\; W_6 , &\text{(E.2e)}
\end{aligned}
$$

corresponding to the six torus directions e_α. Additionally, we divide out the $\mathbb{Z}_2$ symmetry corresponding to

$$\tau \;=\; \frac{1}{2}(e_2+e_4+e_6) \tag{E.3}$$

with a gauge embedding denoted by W (the freely acting Wilson line) where

$$W \;=\; \frac{1}{2}(W_2+W_4+W_6) \;=\; \frac{3}{2}W_2 \;. \tag{E.4}$$

The anomalous space group element reads

$$g_\mathrm{anom} \;=\; (k,\ell;n_1,n_2,n_3,n_4,n_5,n_6) \;=\; (0,0;0,0,1,0,0,0)\,, \tag{E.5}$$

where the boundary conditions of twisted string are

$$X(\tau,\sigma+2\pi) \;=\; \vartheta^k\,\omega^\ell\,X(\tau,\sigma)+n_\alpha e_\alpha \tag{E.6}$$

with ϑ and ω denoting the rotations corresponding to v_1 and v_2. The spectrum is given in table E.1. In addtion there are 37 $G_\mathrm{SM}\times[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{SU}(2)]_\mathrm{hid}$ singlets. In

Label	q_i	$\bar u_i$	$\bar D_i$	D_i	L_i	$\bar L_i$	$\bar e_i$	x_i	$\bar x_i$	y_i	z_i
#	3	3	6	3	9	6	3	5	5	6	6
$\mathrm{SU}(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU}(2)_\mathrm{L}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{U}(1)_Y$	$\frac{1}{6}$	-$\frac{2}{3}$	$\frac{1}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	0	0
SU(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$
SU(2)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
SU(2)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$

Table E.1: The states with their quantum number w.r.t. the SM and the hidden sector.

table E.2 we list the full spectrum. In addition to the states shown there, the spectrum contains the following (untwisted) moduli: the dilaton S, three Kähler moduli T_i and three complex structure moduli U_i.

Table E.2: Spectrum of the model at the orbifold point. The last two columns list the (g)eneral and the (c)onfiguration labels. If there are two labels in one line, this corresponds to the twist parameter $n_1 = 0$ for the first label and $n_1 = 1$ for the second. The two states form a doublet under a D_4 symmetry. In this model the three $\mathbb{Z}_4^R$ charges (corresponding to the three $\mathbb{Z}_2$ orbifold planes) of the respective sectors read: $R(U_1) = (2,0,0)$, $R(U_2) = (0,2,0)$, $R(U_3) = (0,0,2)$, $R(T_{(1,0)}) = (0,1,1)$, $R(T_{(0,1)}) = (1,0,1)$ and $R(T_{(1,1)}) = (1,1,0)$.

sector	irrep	q_{anom}	q_Y	q_X	q_1	q_2	q_3	q_4	q_5	q_6	$q_{\mathbb{Z}_4^R}$	(g)	(c)
U_1	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	−4	72	−88	356	188	−444	60	−4	N_1	ϕ_1
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	−4	$-\frac{1}{2}$	0	−4	0	8	0	0	0	0	$\bar{L}_1$	$\bar{h}_1$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	4	$\frac{1}{2}$	0	4	0	−8	0	0	0	0	L_1	h_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	−4	0	4	−72	88	−356	−188	444	−60	4	N_2	ϕ_2
U_2	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	6	$\frac{1}{2}$	−14	10	−14	62	30	−70	10	−12	L_3	h_3
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	2	0	10	66	−74	286	158	−374	50	12	N_5	ϕ_6
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	−2	0	−10	−66	74	−286	−158	374	−50	−8	N_6	ϕ_7
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	−6	$-\frac{1}{2}$	14	−10	14	−62	−30	70	−10	16	$\bar{L}_3$	$\bar{h}_3$
U_3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	2	0	−14	6	−14	70	30	−70	10	−14	N_3	$\bar{\phi}_4$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	−2	$-\frac{1}{2}$	10	62	−74	294	158	−374	50	10	$\bar{L}_2$	$\bar{h}_2$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	2	$\frac{1}{2}$	−10	−62	74	−294	−158	374	−50	−10	L_2	h_2
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	−2	0	14	−6	14	−70	−30	70	−10	14	N_4	$\bar{\phi}_7$
$T_{(1,0)}^{(*,*,0,0,0,0)}$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	20	2	−2	12	0	0	0	21	$\bar{D}_1$	$\bar{d}_3$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	0	$\frac{1}{2}$	20	2	−2	12	0	0	0	21	L_4	ℓ_3
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	−20	2	2	−20	0	0	0	−19	N_7	n_9
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	2	−1	0	2	0	−4	0	0	0	1	$\bar{E}_1$	$\bar{e}_3$
	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	2	$-\frac{1}{6}$	0	2	0	−4	0	0	0	1	Q_1	q_3
	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	2	$\frac{2}{3}$	0	2	0	−4	0	0	0	1	$\bar{U}_1$	$\bar{u}_3$
$T_{(1,0)}^{(*,*,0,0,1,0)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	0	7	24	−31	140	60	−144	20	8	N_8	ϕ_8
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	−11	48	−57	216	128	−300	40	−10	N_9	$\bar{\phi}_{12}$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	4	$-\frac{1}{2}$	−1	−14	15	−62	−30	74	−10	0	$\bar{L}_4$	$\bar{h}_4$
	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	$\frac{1}{3}$	−1	−14	15	−62	−30	74	−10	0	D_1	δ_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	−8	0	1	10	−15	70	30	−74	10	2	N_{10}	$\bar{\phi}_1$
$T_{(1,0)}^{(*,*,0,0,1,1)}$	$(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$	4	0	−16	−14	16	−62	−34	74	−10	−15	N_{11}	x_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	4	−14	16	−70	−26	66	−10	5	N_{12}	n_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	3	0	−11	−14	16	−70	−30	74	−8	−10	N_{13}	z_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})$	3	0	−21	−14	16	−70	−30	82	−12	−20	N_{14}	y_1
$T_{(1,0)}^{(*,*,1,0,0,0)}$	$(\mathbf{1},\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	2	0	−12	36	−44	182	90	−218	30	−9	N_{15}	$\bar{x}_1$
	$(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$	2	0	8	36	−44	174	98	−226	30	11	N_{16}	x_2
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	0	−2	36	−44	190	90	−226	30	1	N_{17}	n_2
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	−2	36	−44	166	98	−218	30	1	N_{18}	n_3
$T_{(1,0)}^{(*,*,1,0,0,1)}$	$(\mathbf{1},\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	2	0	3	−26	29	−104	−64	148	−20	6	N_{19}	$\bar{x}_2$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	2	0	−17	−26	29	−96	−72	156	−20	−14	N_{20}	$\bar{\phi}_2$
	$(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$	4	0	−7	−26	29	−112	−64	156	−20	−4	N_{21}	x_3
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	13	−26	29	−120	−56	148	−20	16	N_{22}	ϕ_3
$T_{(1,0)}^{(*,*,1,0,1,0)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	4	0	−11	−10	15	−70	−30	82	−10	−8	N_{23}	ϕ_4
	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	11	14	−15	62	30	−82	10	14	$\bar{D}_2$	$\bar{\delta}_1$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	0	$\frac{1}{2}$	11	14	−15	62	30	−82	10	14	L_5	h_4
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	0	21	−48	57	−216	−128	292	−40	24	N_{24}	ϕ_5
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	−4	0	−17	−24	31	−140	−60	152	−20	−14	N_{25}	$\bar{\phi}_3$
$T_{(1,0)}^{(*,*,1,0,1,1)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})$	3	0	−11	−14	16	−70	−30	74	−8	−8	N_{26}	y_2
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	3	0	−21	−14	16	−70	−30	82	−12	−18	N_{27}	z_2
	$(\mathbf{1},\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	0	0	26	14	−16	62	34	−82	10	29	N_{28}	$\bar{x}_3$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	0	6	14	−16	70	26	−74	10	9	N_{29}	n_4
$T_{(0,1)}^{(n_1,0,*,*,0,0)}$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	$-\frac{1}{3}$	13	5	−9	47	15	−35	5	13	$\bar{D}_3$, $\bar{D}_4$	$\bar{d}_2$, $\bar{d}_1$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	1	$\frac{1}{2}$	13	5	−9	47	15	−35	5	13	L_6, L_7	ℓ_2, ℓ_1
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	5	0	−27	5	−5	15	15	−35	5	−27	N_{30}, N_{36}	n_5, n_6
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	3	−1	−7	5	−7	31	15	−35	5	−7	$\bar{E}_2$, $\bar{E}_3$	$\bar{e}_2$, $\bar{e}_1$
	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	3	$-\frac{1}{6}$	−7	5	−7	31	15	−35	5	−7	Q_2, Q_3	q_2, q_1
	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	3	$\frac{2}{3}$	−7	5	−7	31	15	−35	5	−7	$\bar{U}_2$, $\bar{U}_3$	$\bar{u}_2$, $\bar{u}_1$

sector	irrep	q_{anom}	q_Y	q_X	q_1	q_2	q_3	q_4	q_5	q_6	$q_{\mathbb{Z}_4^R}$	(g)	(c)
$T_{(0,1)}^{(n_1,0,*,*,1,0)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	0	14	21	-24	105	45	-109	15	14	$N_{31},\, N_{37}$	$\bar{\phi}_5,\, \bar{\phi}_6$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	3	0	-4	45	-50	181	113	-265	35	-4	$N_{32},\, N_{38}$	$\phi_9,\, \phi_{10}$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	3	$-\frac{1}{2}$	6	-17	22	-97	-45	109	-15	6	$\bar{L}_5,\, \bar{L}_6$	$\bar{h}_5,\, \bar{h}_6$
	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	3	$\frac{1}{3}$	6	-17	22	-97	-45	109	-15	6	$D_2,\, D_3$	$\delta_2,\, \delta_3$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-9	0	8	7	-8	35	15	-39	5	8	$N_{33},\, N_{39}$	$\phi_{11},\, \phi_{12}$
$T_{(0,1)}^{(n_1,0,*,*,1,1)}$	$(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$	3	0	-9	-17	23	-97	-49	109	-15	-9	$N_{34},\, N_{40}$	$x_4,\, x_5$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	3	0	11	-17	23	-105	-41	101	-15	11	$N_{35},\, N_{41}$	$\bar{n}_1,\, \bar{n}_2$
$T_{(1,1)}^{(n_1,0,0,0,*,*)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	5	0	3	-55	65	-251	-143	339	-45	4	$N_{42},\, N_{51}$	$\phi_{13},\, \phi_{14}$
	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-3	$-\frac{1}{3}$	-13	-7	9	-43	-15	35	-5	-12	$\bar{D}_5,\, \bar{D}_6$	$\bar{\delta}_2,\, \bar{\delta}_3$
	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	-3	$\frac{1}{2}$	-13	-7	9	-43	-15	35	-5	-12	$L_8,\, L_9$	$h_5,\, h_6$
$T_{(1,1)}^{(n_1,0,0,1,*,*)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})$	6	0	13	7	-8	35	15	-43	7	14	$N_{43},\, N_{52}$	$y_3,\, y_5$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	6	0	3	7	-8	35	15	-35	3	4	$N_{44},\, N_{53}$	$z_3,\, z_5$
	$(\mathbf{1},\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	5	0	8	7	-8	27	19	-35	5	9	$N_{45},\, N_{54}$	$\bar{x}_4,\, \bar{x}_5$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	5	0	-12	7	-8	35	11	-27	5	-11	$N_{46},\, N_{55}$	$n_7,\, n_8$
$T_{(1,1)}^{(n_1,0,1,0,*,*)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	5	0	3	3	-7	35	15	-43	5	6	$N_{47},\, N_{56}$	$\bar{\phi}_8,\, \bar{\phi}_{10}$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-3	0	-17	3	-7	35	15	-27	5	-14	$N_{48},\, N_{57}$	$\bar{\phi}_9,\, \bar{\phi}_{11}$
$T_{(1,1)}^{(n_1,0,1,1,*,*)}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	6	0	13	7	-8	35	15	-43	7	16	$N_{49},\, N_{58}$	$z_4,\, z_6$
	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})$	6	0	3	7	-8	35	15	-35	3	6	$N_{50},\, N_{59}$	$y_4,\, y_6$

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